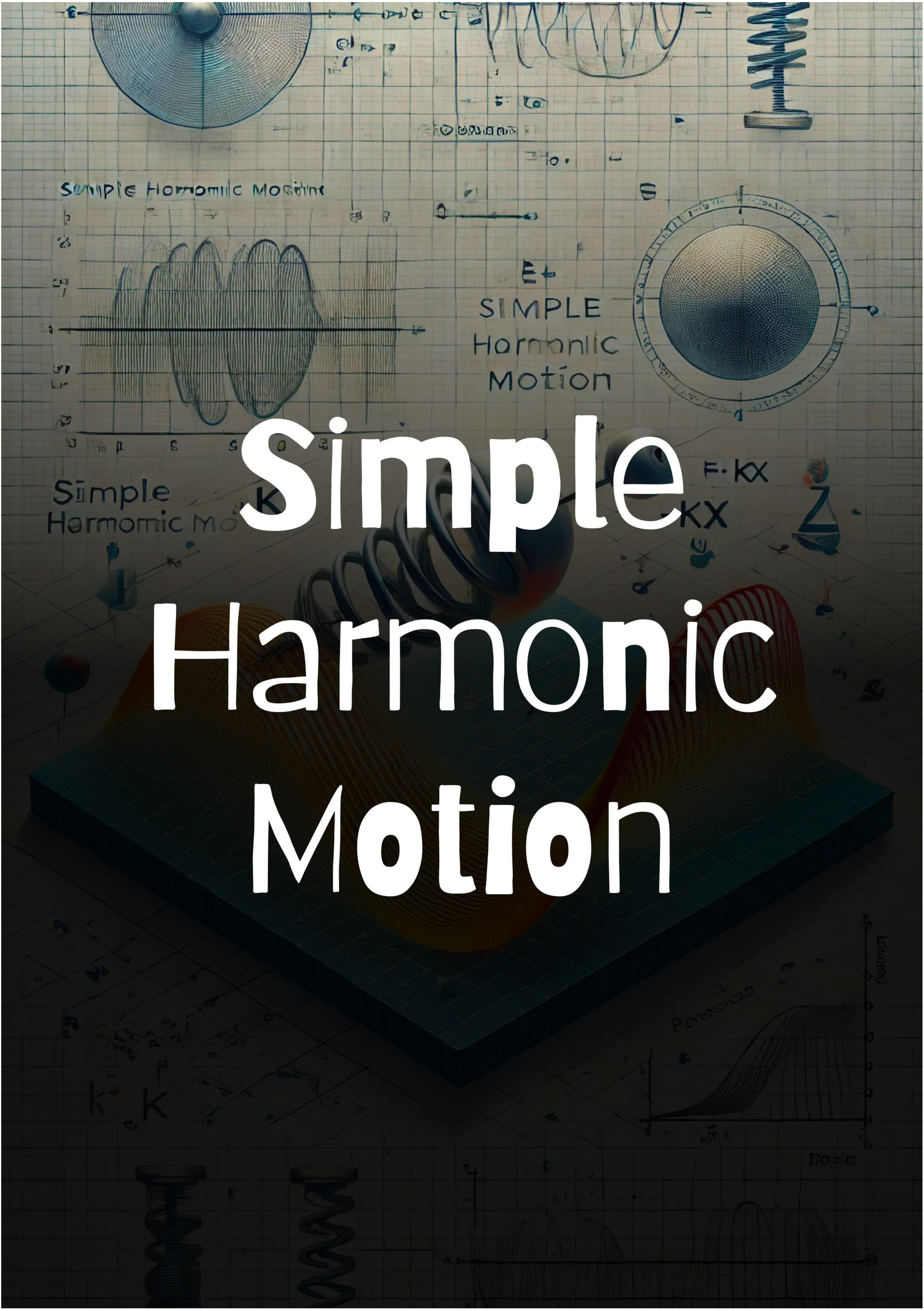


Simple Harmonic Motion

Simple Harmonic Motion (SHM)

SECOND ORDER SHM DIFFERENTIAL EQUATION

Any equation of the form $\frac{d^2x}{dt^2} = -\omega^2(x - x_0)$ is called an SHM differential equation. Here ω and x_0 must be constants. They are respectively called angular frequency (ω) and mean position (x_0).

Here x is called the position of the particle and t is time.

Que.) For the shown spring mass system, find the angular frequency and the mean position taking origin at the natural length position and downward as positive direction.

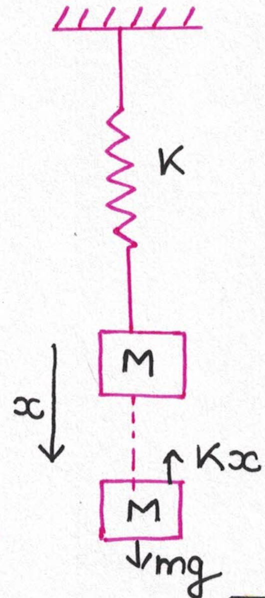
$$Mg - Kx = M\left(\frac{d^2x}{dt^2}\right)$$

$$\frac{d^2x}{dt^2} = -\frac{K}{M}\left(x - \frac{Mg}{K}\right)$$

$$\therefore \omega = \sqrt{\frac{K}{M}}$$

and

$$x_0 = \frac{Mg}{K}$$



SOLUTION OF SHM DIFFERENTIAL EQUATION

The solution to the general SHM differential equation is given by:-

1. $x = x_0 + A\sin(\omega t + \phi)$
2. $x = x_0 + A\cos(\omega t + \phi)$

where A & ϕ are arbitrary constants.

VERIFICATION OF SOLUTION-1

If $x = x_0 + A\sin(\omega t + \phi)$

$$\frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -A\omega^2 \sin(\omega t+\phi)$$

$$\frac{d^2x}{dt^2} = \frac{-\cancel{A}\omega^2 (x-x_0)}{\cancel{A}}$$

$$\frac{d^2x}{dt^2} = -\omega^2(x-x_0)$$

VERIFICATION OF SOLUTION - 2

If $x = x_0 + A\cos(\omega t+\phi)$

$$\frac{dx}{dt} = -A\omega \sin(\omega t+\phi)$$

$$\frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t+\phi)$$

$$\frac{d^2x}{dt^2} = \frac{-\cancel{A}\omega^2 (x-x_0)}{\cancel{A}}$$

$$\frac{d^2x}{dt^2} = -\omega^2(x-x_0)$$

FINDING THE PARTICULAR SOLUTION FROM THE GENERAL SOLUTION

To solve for two arbitrary constants of the general solution, we need two initial conditions. Typically, x @ $t=0$ and v @ $t=0$. Using these two conditions we can find the particular solution by substituting these conditions in the general solutions.

Que.) The general Sol^n of an SHM differential eq^n is

$$x = 5 + A\sin\left(\frac{\pi}{6}t+\phi\right)$$

Find particular sol^n if @ $t=0$, $u=0$ & $v=0$

$$0 = 5 + A\sin\phi$$

$$v = \frac{A\pi}{6}\cos\left(\frac{\pi t}{6}+\phi\right)$$

$$0 = \frac{A\pi}{6}\cos(\phi)$$

$$\sin\phi = -\frac{5}{A}$$

$$\cos\phi = 0$$

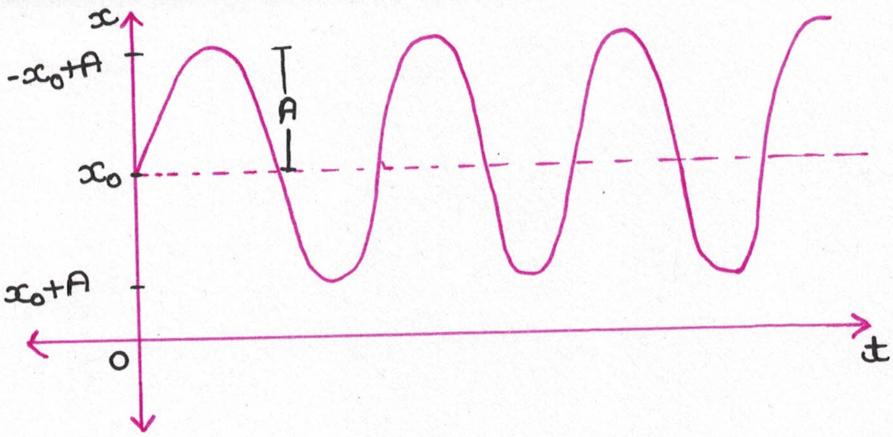
$$\tan\phi = \pm\infty$$

$$\phi = \pm\frac{\pi}{2}$$

we choose that value of ϕ for which A comes positive.

$$\phi = -\frac{\pi}{2}, \quad A = 5$$

PLOT OF THE SHM SOLUTION



NOTE: Here x_0 is called the mean position because the particle oscillates equally above and below this position. The term A is called the amplitude of oscillation.

PHASOR REPRESENTATION OF SHM

Consider the general solution of SHM differential eq^n

$$x = x_0 + A\cos(\omega t + \phi)$$

If we imagine a vector whose tail is at x_0 and length equal to A and rotating with an angular velocity ω, then the coordinate of the shadow of the tip of the rotating vector is also given as $x = x_0 + A\cos(\omega t + \phi)$.

Thus any SHM can be visualized as a shadow of an imaginary rotating vector centered at the mean position. This imaginary vector is called the phasor of the SHM.

- It is clear from the phasor diagram that the particle will repeat its configuration after a time $2\pi/\omega$. This is called time period of oscillation.

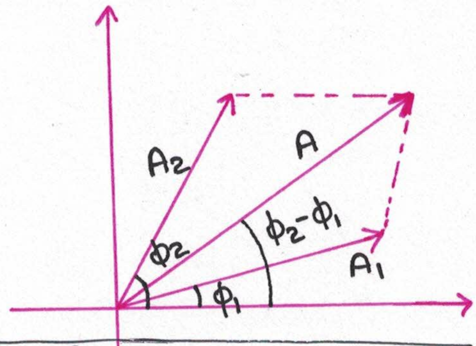
  Mathematically, $\boxed{T = \dfrac{2\pi}{\omega}}$

- Repetition of configuration means repetition of position as well as velocity (including the directions)

PHASOR ADDITION OF SHM

$x_1 = A_1 \sin(\omega t + \phi_1)$

$x_2 = A_2 \sin(\omega t + \phi_2)$



$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2-\phi_1)}$$

$$\tan\phi = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$$

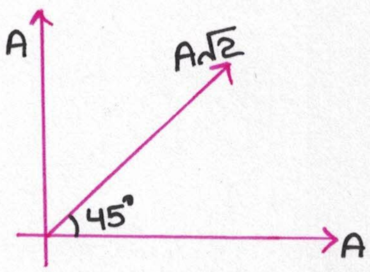
$$x_1 + x_2 = A\sin(\omega t + \phi)$$

Que.) Add the following SHM —

(i) $x_1 = A\sin(\omega t)$

$x_2 = A\cos(\omega t) = A\sin\left(\omega t + \frac{\pi}{2}\right)$

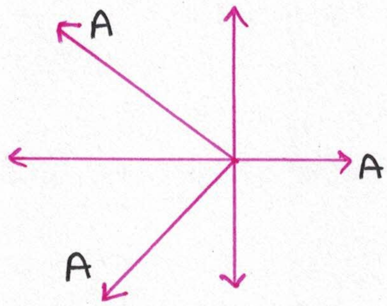
$x_1 + x_2 = A\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$



(ii) $x_1 = A\sin(\omega t)$

$x_2 = A\sin\left(\omega t + \frac{2\pi}{3}\right)$

$x_3 = A\sin\left(\omega t + \frac{4\pi}{3}\right)$



$\therefore x_1 + x_2 + x_3 = 0$

3 METHODS OF DEVELOPING SHM EQ^N

SHM eq^n can be developed using the following method:

(1) By Force Method $\left(F = m\frac{d^2x}{dt^2}\right)$

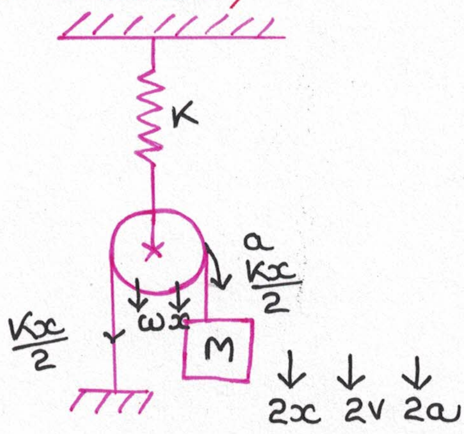
(2) By Torque Method $\left(\tau = I\frac{d^2\theta}{dt^2}\right)$

(3) $\left[\frac{d}{dt}(COME) = 0\right]$ By Energy Method

FORCE METHOD

In this method we write the Newton's Second law at the generalised coordinates of a system.

Que.) Find the time period of the system. (Pulley is massless.)

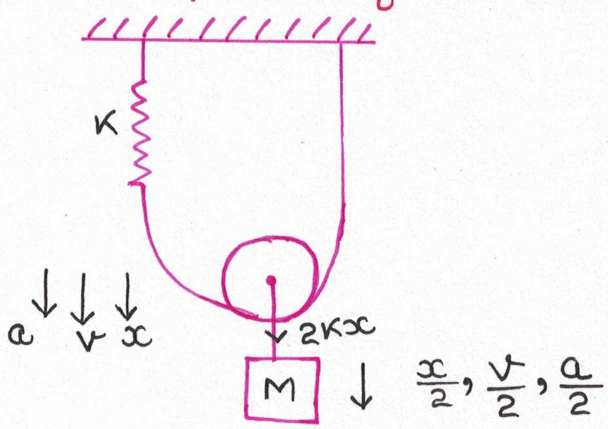


$$Mg - \frac{Kx}{2} = M \times \frac{2d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{-K}{4M}\left(x - \frac{2Mg}{K}\right)$$

$$T = 4\pi\sqrt{\frac{M}{K}}$$

Que.) Find time period of Oscillation.



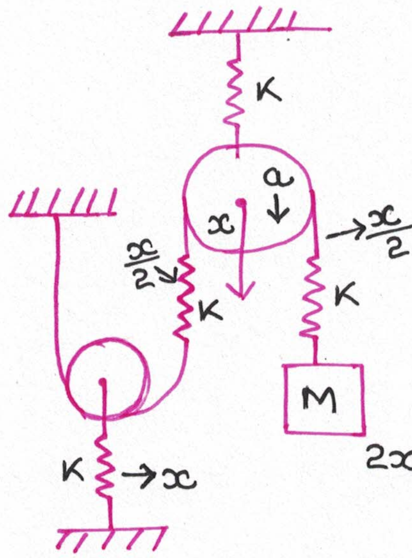
$$Mg - 2Kx = \frac{M}{2}\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{4K}{M}\left(x - \frac{Mg}{2K}\right)$$

$$T = 2\pi\sqrt{\frac{M}{4K}}$$

Mean position of block is $\frac{Mg}{4K}$

Que.) The shown system is released from rest from natural length config. of spring. Find
(a) Time point of oscillation.
(b) Mean position of the block.
(c) Amplitude of oscillation.



$$2x+\frac{x}{2}+\frac{x}{2}+2x = 5x,\ 5v,\ 5a$$

$$Mg - Kx = 5M\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{-Kx}{10M} + \frac{g}{5}$$

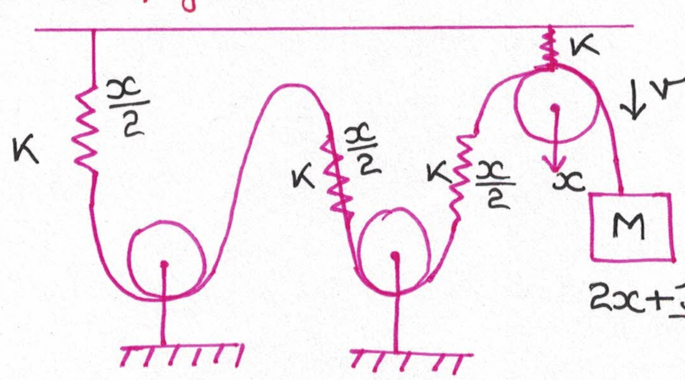
$$\frac{d^2x}{dt^2} = \frac{-K}{10M}\left(x - \frac{2Mg}{K}\right)$$

$$T = 2\pi\sqrt{\frac{10M}{K}}$$

$$x = \frac{2Mg}{K}$$

$\therefore$ Mean position of block $5\times\frac{2Mg}{K} = \frac{10Mg}{K}$ below the initial position of the block.

Que.) Shown system is released from rest from natural length configuration.



$$2x+\frac{3x}{2} = \frac{7x}{2}\ \frac{\omega}{2}$$

$$Mg - \frac{Kx}{2} = M\frac{d^2x}{dt^2}\times\frac{7}{2}$$

$$\frac{d^2x}{dt^2} = \frac{-K}{7M}\left(x - \frac{2gM}{K}\right)$$

Mean position of block $= \frac{7gM}{K} \left\{\frac{7}{2} \times \frac{2gM}{K}\right\}$

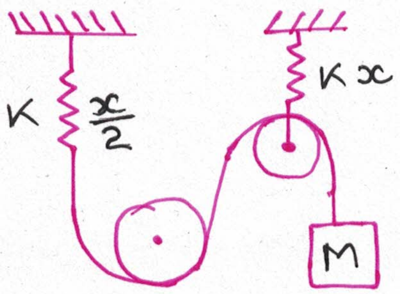
Time period $= 2\pi\sqrt{\frac{7M}{K}}$

NOTE: Mean position can be quickly found by equating the force on any massive object to zero. This can be explained as follows:

@ $x = x_0$, $\frac{d^2x}{dt^2} = 0$

$F = 0$

Que.)



$2v + \frac{x}{2} = \frac{5x}{2}, \frac{5v}{2}$

Mean position $= \frac{Kx}{2} = Mg$ (@ $x = x_0$)

$$x_0 = \frac{2Mg}{K}$$

$$\therefore \frac{5x}{2} = \frac{5Mg}{K}$$

TORQUE METHOD

This method is similar to force method except that here we make eqn. as follows:

$$\tau_{IAUR} = I_{IAUR}\frac{d^2\theta}{dt^2}$$

or $$\tau_{cm} = I_{cm}\frac{d^2\theta}{dt^2}$$

SHM D.E. FOR ANGULAR OSCILLATIONS

$$\frac{d^2\theta}{dt^2} = -\omega^2(\theta - \theta_0)$$

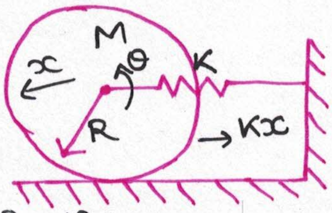
GENERAL SOLUTIONS

$$\theta = \theta_0 + A\sin(\omega t + \phi)$$

or $$\theta = \theta_0 + A\cos(\omega t + \phi)$$

NOTE: Angular frequency of oscillation must never be confused with the angular velocity of the rigid body.

Que.) Find the time pd. of oscillation of the shown cylinder. (given – no slipping)



$$-KxR = \frac{3}{2}MR^2 \cdot \frac{d^2\theta}{dt^2}$$

$$\frac{dx}{dt} = R\frac{d\theta}{dt} \quad \text{(pure rolling)}$$

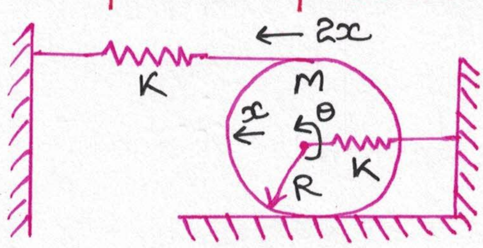
$$x = R\theta$$

$$-KR^2\theta = \frac{3}{2}MR^2 \cdot \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = \frac{-2K}{3M}\theta$$

$$T = 2\pi\sqrt{\frac{3M}{2K}}$$

Que.) Repeat the previous problem for the shown situation.



$$-(2Kx \times 2R + Kx \times R) = \frac{3}{2}MR^2 \frac{d^2\theta}{dt^2}$$

$$-5KR^2\theta = \frac{3}{2}MR^2 \frac{d^2\theta}{dt^2}$$

$$\omega = \sqrt{\frac{10K}{3M}}$$

$$T = 2\pi\sqrt{\frac{3M}{10K}}$$

PHYSICAL PENDULUM

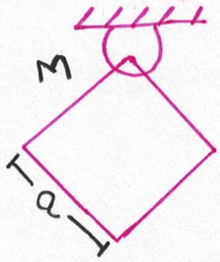
$$-MgD\sin\theta = I\frac{d^2\theta}{dt^2}$$

If θ is small then $\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} = -\frac{MgD\theta}{I}$$

$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

Que.) Find time period for the following situation.



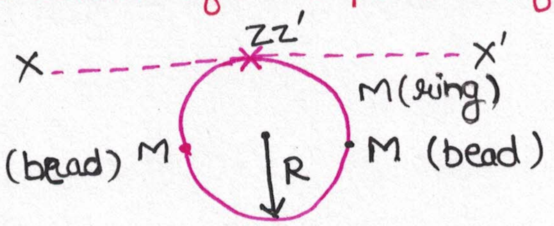
$$I = \frac{2}{3}Ma^2$$

$$D = \frac{a}{\sqrt{2}}$$

$$T = 2\pi\sqrt{\frac{I}{MgD}} = 2\pi\sqrt{\frac{2}{3}\frac{\cancel{M}R^2}{\cancel{M}g\,a}\sqrt{2}}$$

$$= 2\pi\sqrt{\frac{2}{3}\frac{a\sqrt{2}}{g}}$$

Que.) Find the ratio of time period of oscillations about XX' & ZZ'.



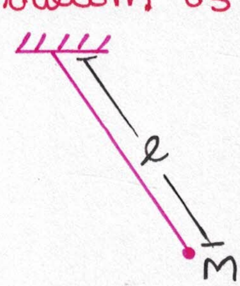
$$\frac{T_{XX'}}{T_{ZZ'}} = \sqrt{\frac{I_{XX'}}{I_{ZZ'}}}$$

$$= \sqrt{\frac{3/2\,MR^2 + 2MR^2}{2MR^2 + M(R\sqrt{2})^2 \times 2}} = \sqrt{\frac{7/2\,MR^2}{6MR^2}}$$

Que.) Prove that time period of simple pendulum is $2\pi\sqrt{l/g}$.

$$T = 2\pi\sqrt{\frac{M\ell^{\cancel{2}}}{Mg\cancel{\ell}}}$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$



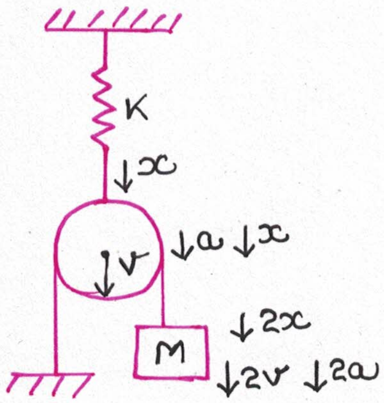
(1) Write the COME equation at the general configuration of the system.

(2) Differentiate the entire equation w.r.t. time i.e. $\frac{d}{dt}(\text{COME}) = 0$

(3) Cancel a suitable factor throughout
(Usually ~~to form SHM D.E~~ velocity or angular velocity)

(4) Rearrange to form SHM D.E.

Que.)



$$\frac{1}{2}Kx^2 - 2Mgx + \frac{1}{2}M(2v)^2 = C$$

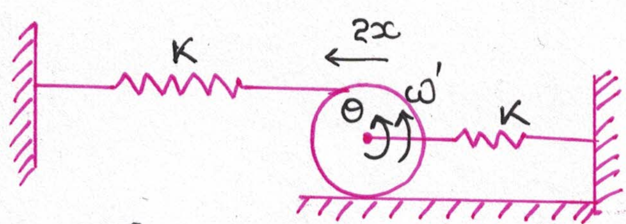
$$Kx\cancel{v} - 2Mg\cancel{v} + 4M\cancel{v}\frac{d^2x}{dt^2} = 0 \qquad \text{(differentiating)}$$

$$\frac{d^2x}{dt^2} = \frac{-K}{4M}x + 2g$$

$$\frac{d^2x}{dt^2} = \frac{-K}{4M}\left(x - \frac{2Mg}{K}\right)$$

$$2x = \frac{4Mg}{K}$$

Que.)



$$\frac{1}{2}K(2x)^2 + \frac{1}{2}K(x)^2 + \frac{1}{2}\cdot\frac{3}{2}MR^2\omega'^2 = C$$

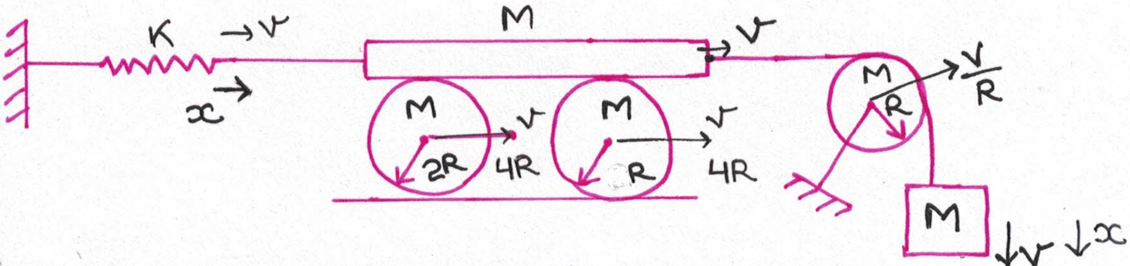
$$\frac{5}{2}KR^2\theta^2 + \frac{3}{4}MR^2\omega'^2 = C \qquad [x = R\theta]$$

$$\frac{5}{2}KR^2\theta\omega' + \frac{3}{2}MR^2\omega'\frac{d^2\theta}{dt^2} = 0$$

$$\frac{d^2\theta}{dt^2} = \frac{-10K\theta}{3M}$$

~~$\omega = \sqrt{10K}$~~ $\rightarrow \omega = \sqrt{\frac{10K}{3M}}$

Que.)



$$\frac{1}{2}Kx^2+\frac{1}{2}Mv^2+\frac{1}{2}M\times\frac{3}{2}\frac{4R^2v^2}{16R^2}+\frac{1}{2}\times\frac{3}{2}MR^2\cdot\frac{v^2}{4R^2}$$
$$+\frac{1}{2}\times\frac{MR^2}{2}\times\frac{v^2}{R^2}+\frac{1}{2}Mv^2.\ Mgx = \text{Constant}$$

$$\frac{1}{2}Kx^2+Mv^2\left(\frac{1}{2}+\frac{3}{16}+\frac{3}{16}+\frac{1}{4}+\frac{1}{2}\right)-Mgx = \text{Const.}$$

$$\frac{1}{2}Kx^2+\frac{26}{16}Mv^2-Mgx = \text{Const.}$$

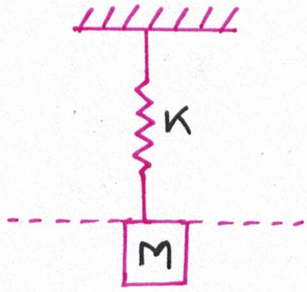
$$Kxv+\frac{52}{16}Mv.\frac{d^2x}{dt^2}-Mg.v=0$$

$$\frac{d^2x}{dt^2}=\frac{Mg-Kx}{52M}\times 16$$

$$\frac{d^2x}{dt^2}=\frac{-16K}{52M}\left(x-\frac{Mg}{K}\right)$$

$$T=2\pi\sqrt{\frac{13M}{4K}}$$

VARIATION OF K.E. & P.E. IN SHM WITH POSITION



$$x = x_0 + A\sin\omega t$$
$$v = A\omega\cos\omega t$$

$$K.E. = \frac{1}{2}MA^2\omega^2\cos^2\omega t$$

$$P.E. = \frac{1}{2}Kx^2 - Mgx$$

$$\cos^2\omega t = 1-\sin^2\omega t$$

$$K.E. = \frac{1}{2}MA^2\frac{K}{M}\left[1-\left(\frac{x-x_0}{A}\right)^2\right] \qquad \left(\omega=\sqrt{\frac{K}{M}}\right)$$

$$=\frac{1}{2}K\left[A^2-(x-x_0)^2\right] \qquad \left(x_0=\frac{Mg}{K}\right)$$

$$=\frac{1}{2}KA^2-\frac{1}{2}Kx^2-\frac{1}{2}Kx_0^2+Kxx_0$$

$$K.E. + P.E. = \frac{1}{2}KA^2-\frac{1}{2}Kx_0^2$$

$$T.E. = \frac{1}{2}K\left(A^2-x_0^2\right) = \text{Constant}$$

SPEED IN SHM AS A FUNCTION OF POSITION

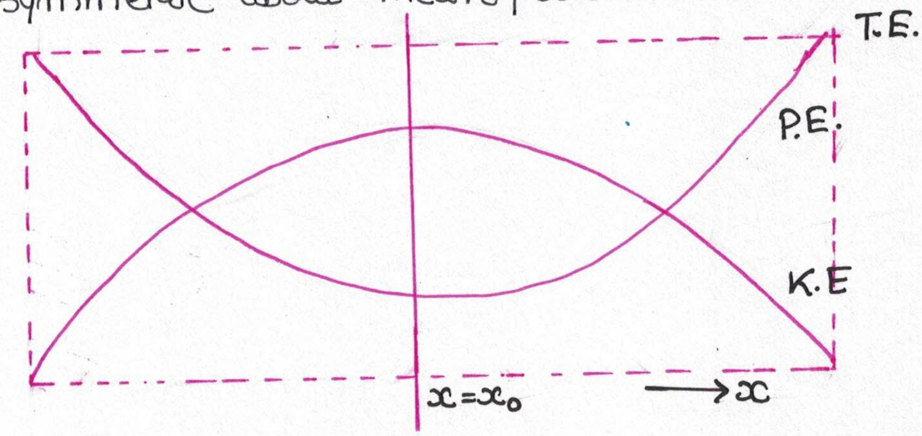
$x = x_0 + A\sin(\omega t + \phi)$

$v = A\omega\cos(\omega t + \phi)$

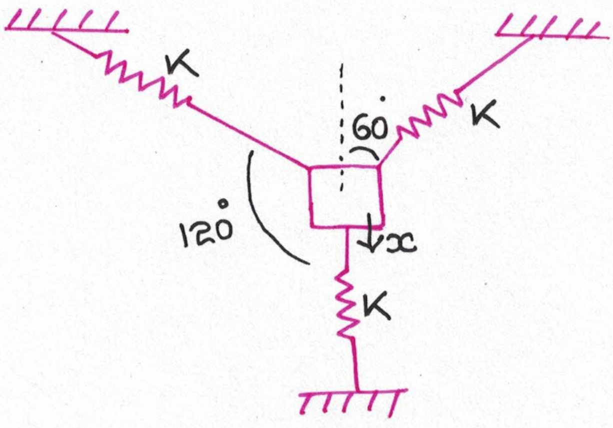
$v = A\omega\sqrt{1 - \left(\frac{x - x_0}{A}\right)^2}$

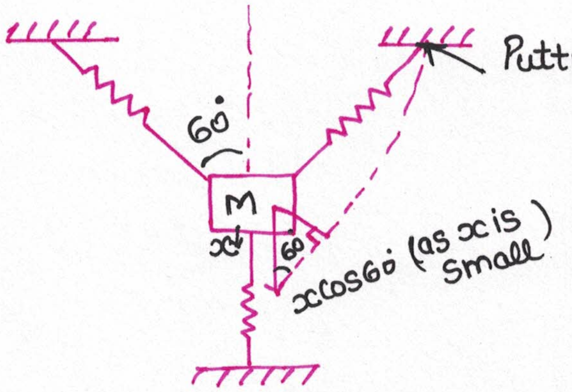
$$\boxed{v = \omega\sqrt{A^2 - (x - x_0)^2}}$$

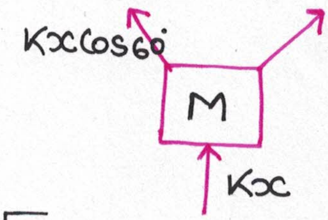
NOTE: We see that speed is maximum at $x = x_0$ and speed is minimum when $|x - x_0| = A$. We can also see that motion is symmetric about mean position.



Que.) The shown block is in equilibrium in the vertical plane. Find the time period of small vertical oscillations of the block.







$$-\frac{3}{2}Kx = M\frac{d^2x}{dt^2}$$

[Change in forces is represented because it was already in equilibrium]

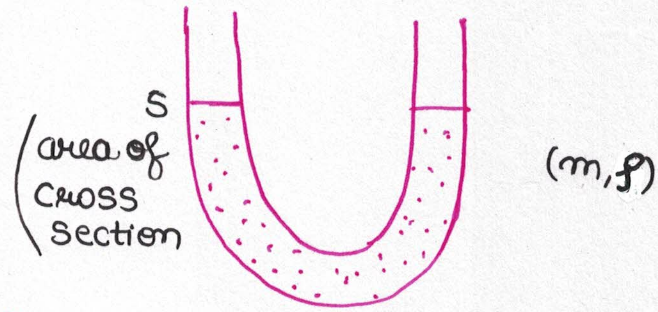
Que.) Find the time period of oscillation of water in the shown situation.

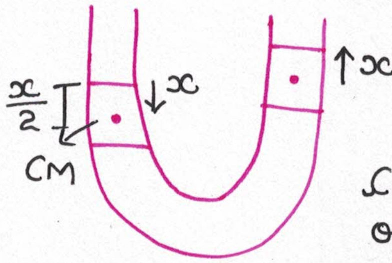
$$\rho S x g x + \frac{1}{2}mv^2 = C$$

$$2\rho S x v + mv\frac{d^2x}{dt^2} = 0$$

$$\frac{d^2x}{dt^2} = \frac{-2\rho S x g}{m}$$

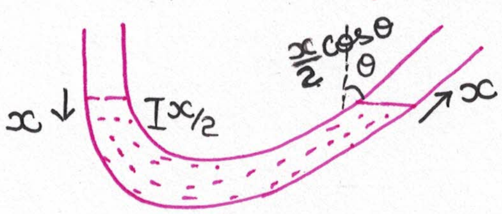
$$T = 2\pi\sqrt{\frac{m}{2\rho S g}}$$





Change in position of mass $= \frac{x}{2} + \frac{x}{2} = x$

Que.) Repeat the previous problem for the shown situation.

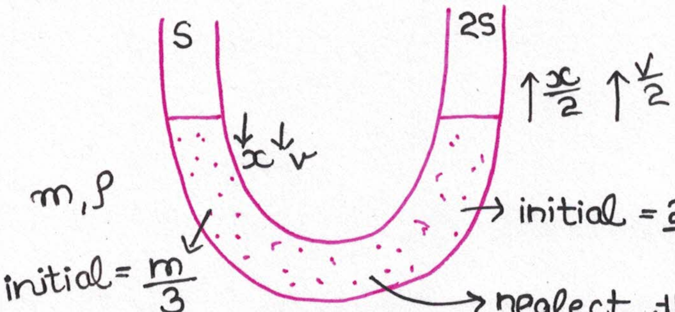


$$\rho s g x \times \left(\frac{x}{2} + \frac{x}{2}\cos\theta\right) + \frac{1}{2}mv^2 = 0$$

$$\rho s(1+\cos\theta)xv + mv\frac{d^2x}{dt^2} = 0$$

$$T = 2\pi\sqrt{\frac{m}{\rho s g(1+\cos\theta)}}$$

Que.) Will there be SHM or not?



final = $\frac{m}{3} - \rho s x$

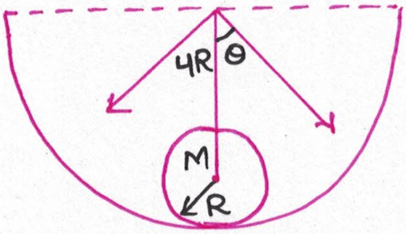
$$\rho s x . g + \left(\frac{3x}{4}\right) + \frac{1}{2}\left(\frac{m}{3} - \rho s x\right)v^2 + \frac{1}{2}\left(\frac{2m}{3} + \rho s x\right)\left(\frac{v}{2}\right)^2 = C$$

$$(am - bx)v^2 + (cm + dx)v^2 = C$$

$$v^2(-bv) + (am - bx)\,2v.\frac{dv}{dt} + -----$$

Clearly, v is not getting cancel out throughout the eqn., therefore it is not an SHM.

Que.) Find the time period of oscillation of cylinder assuming no slipping.

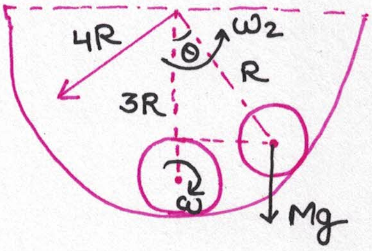


$$\omega_1 3R = \omega' R$$

$$\omega' = 3\omega_1$$

$$\phi = 3\theta$$

angular displacement of cylinder



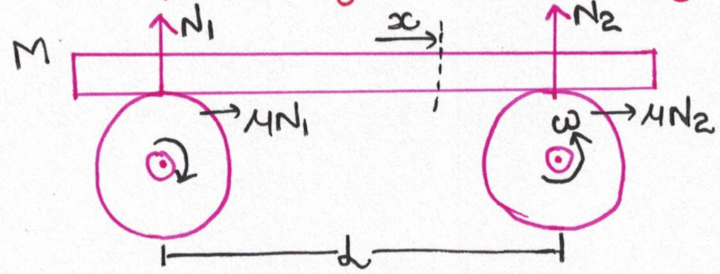
$$-MgR\sin\theta = \frac{3}{2}MR^2\frac{d^2\phi}{dt^2}$$

$$-\cancel{M}g\cancel{R}\sin\frac{\phi}{3} = \frac{3}{2}\cancel{M}R^{\cancel{2}}\frac{d^2\phi}{dt^2}$$

$$\frac{-2g}{9R}\phi = \frac{d^2\phi}{dt^2}$$

$$T = 2\pi\sqrt{\frac{9R}{2g}}$$

Que.) A plank is placed on two rough rollers as shown. Find out the time period of oscillation of the plank.



$N_1 + N_2 = Mg$

$N_1\left(\frac{l}{2}+x\right) = N_2\left(\frac{l}{2}-x\right)$

$N_2 = \frac{Mg}{2}\left(1+\frac{2x}{l}\right)$

$N_1 = \frac{Mg}{2}\left(1-\frac{2x}{l}\right) \quad -\left(\mu\left(N_2-N_1\right)\right) = m\frac{d^2x}{dt^2}$

$\frac{-\mu Mgx}{l} = m\frac{d^2x}{dt^2}$

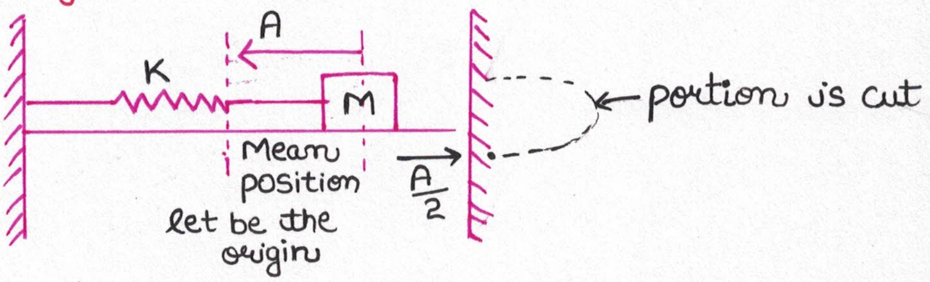
$T = 2\pi\sqrt{\frac{l}{2\mu g}}$

Here, we can directly find out relation b/w N_1 & N_2 because here torque about any point is zero.

$\tau_0 = m(\vec{r}_{cm} \times \vec{a}_{cm}) + I_{cm}\alpha \rightarrow$ zero

MODIFIED SHM

Que.) A block is used to compress the spring from its mean position through a distance A, and there is a perfectly elastic wall and there is a distance $A/2$ on the other side.
Find the time period of oscillation of the block which is tied to spring.



$x = x_0 + A\sin(\omega t + \phi)$

@ $t = 0$, $x = -A$, $v = 0$

$-A = A\sin\phi$

$0 = A\omega\cos\phi$

$\therefore A = A$

$\phi = -\frac{\pi}{2}$

$x = A\sin\left(\omega t - \frac{\pi}{2}\right)$

@ $t = T_0$, $x = +\frac{A}{2}$

$$\frac{A}{2} = A\sin\left(\omega T_\omega - \frac{\pi}{2}\right)$$

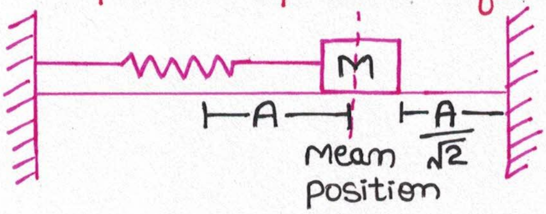
$$\cos(\omega T_\omega) = -\frac{1}{2}$$

$$\omega T_\omega = \frac{2\pi}{3}$$

$$T_\omega = \frac{2\pi}{3\omega}$$

$$T = 2T_\omega = \frac{4\pi}{3\omega} \qquad \left(\omega = \sqrt{\frac{K}{M}}\right)$$

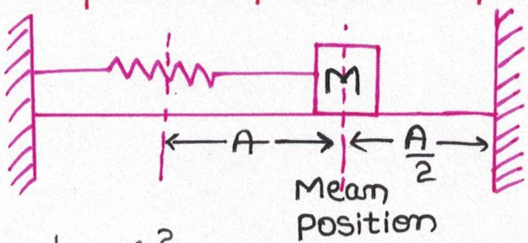
Que.) Repeat the previous problem for shown case.



$$@\,T = T_\omega \ , \ x = \frac{A}{\sqrt{2}}$$

$$\frac{A}{\sqrt{2}} = A\sin\left(\omega t_\omega - \frac{\pi}{2}\right)$$

Que.) Repeat the previous problem if block is not tied to spring.



$$\frac{1}{2}KA^2 = \frac{1}{2}m v_m^2 \qquad (v_m \rightarrow \text{velocity at mean position})$$

$$\sqrt{\frac{K}{M}}\,A = V_m = A\omega$$

$$A = A$$

$$\phi = -\frac{\pi}{2}$$

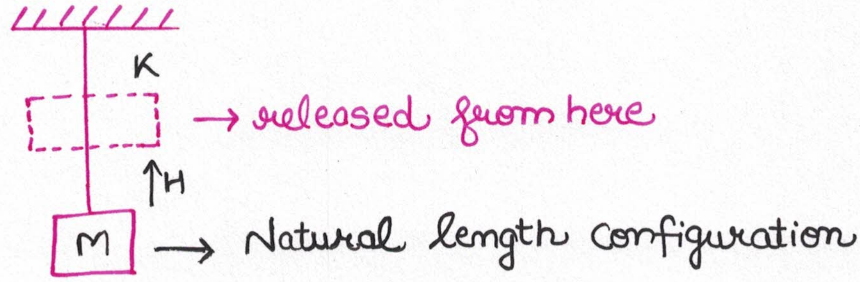
$$T_{mean} = \frac{2\pi}{\omega} \times \frac{1}{4} = \frac{\pi}{2\omega}$$

$$T_{additional} = \frac{A/2}{V_{mean}} = \frac{A/2}{A\omega} = \frac{1}{2\omega}$$

$$\therefore \text{Total time} = 2\left(\frac{\pi}{2\omega} + \frac{1}{2\omega}\right)$$

$$= \frac{\pi + 1}{\omega}$$

Que)



$$x = A\sin(\omega t+\phi)+x_0$$

@ $t=0$, $v=\sqrt{2gH}$, $x=0$

$$A\omega\cos\phi = \sqrt{2gH}$$

$$\cos\phi = \frac{\sqrt{2gH}}{\omega A}$$

$$0 = x_0 + A\sin\phi$$

$$A\sin\phi = -x_0 = -\frac{Mg}{K}$$

$$\sin\phi = -\frac{Mg}{KA}$$

$$\tan\phi = -\frac{Mg\omega}{K\sqrt{2gH}}$$

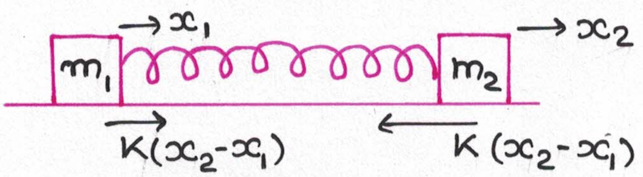
we can also find A by using COME

$$MgH = \frac{1}{2}K(A+x_0)^2 - Mg(A+x_0)$$

Thus we can find time when $x = x_0+A$

Then add $\sqrt{2gH}$ and multiply it by 2.

TWO BODY PROBLEM



$$m_2\frac{d^2x_2}{dt^2} = -K(x_2-x_1) \quad \text{—(1)}$$

$$m_1\frac{d^2x_1}{dt^2} = K(x_2-x_1) \quad \text{—(2)}$$

$$\frac{(1)}{m_2} - \frac{(2)}{m_2}$$

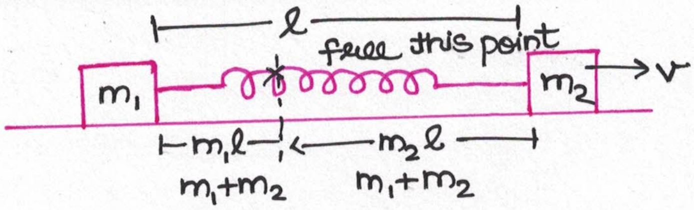
$$\frac{d^2}{dt^2}(x_2-x_1) = \left(-\frac{K}{m_1}-\frac{K}{m_2}\right)(x_2-x_1)$$

$$\frac{d^2x}{dt^2} = -K\left(\frac{m_1+m_2}{m_1m_2}\right)x$$

μ(reduced mass) $= \dfrac{m_1 m_2}{m_1+m_2}$ (always $\mu < 1$)

$$T = 2\pi\sqrt{\frac{\mu}{K}}$$

ANOTHER METHOD TO SOLVE SUCH PROBLEMS (from centre of mass frame)



Now consider it as a wall (the point of COM)

we know that if spring is cut then K is inversely proportional to the length.

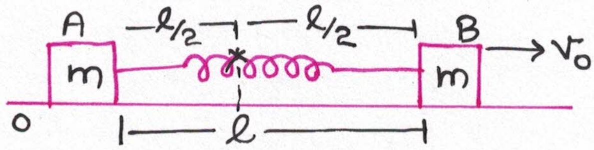
For eg: If length is half then K becomes 2 times.

$$K_2 = \left(\frac{m_1+m_2}{m_1}\right)K$$

$$T = 2\pi\sqrt{\frac{m_2}{K_2}}$$

$$= 2\pi\sqrt{\frac{m_1 m_2}{(m_1+m_2)K}}$$

Que.) Find coordinates of block A.



$x = x_0 + A\sin(\omega t + \phi)$

For block (A) in Centre of mass frame

@ $t=0$, $x_0 = \dfrac{-l}{2}$, $x = \dfrac{-l}{2}$, $v = \dfrac{-v}{2}$

$$\frac{-l}{2} = \frac{-l}{2} + A\sin\phi$$

$$\sin\phi = 0 \quad —(1)$$

$$\frac{-v_0}{2} = A\omega\cos\phi$$

$$\phi = \pi$$

$A = \dfrac{v_0}{2\omega}$, where $\omega = \sqrt{\dfrac{K}{\mu}} = \sqrt{\dfrac{K}{2m}}$

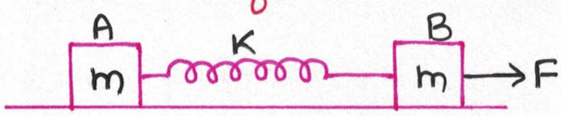
$$x = -\frac{l}{2} + \frac{v_0}{2\omega}\sin(\omega t + \pi)$$ (from centre of mass frame)

From ground frame,

$$x = \frac{l}{2} + \frac{v_0 t}{2} - \frac{l}{2} + \frac{v_0}{2\omega}\sin(\omega t + \pi)$$

$$x_A = \frac{v_0 t}{2} + \frac{v_0}{2\omega}\sin(\omega t + \pi)$$

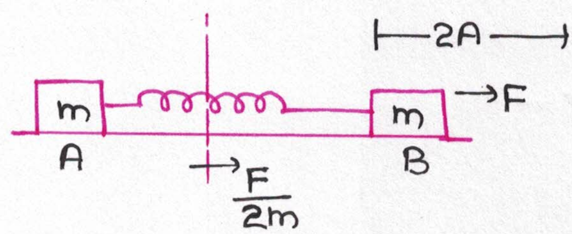
Que.) The shown system starts from rest. Find
(i) The amplitude of the blocks as seen from C.M.
(ii) Position of blocks as seen from CM
(iii) Position of block as seen from C.M.



(Chime on B)

(i) $\frac{F}{2} \times 2A = \frac{1}{2}(2K)(2A)^2$

$$A = \frac{F}{4K}$$



(ii) $x_B = x_0 + A\sin(\omega t + \phi)$

@ $t = 0$

$$x_0 = \frac{\alpha}{2} + \frac{F}{4K}$$ (from C.M.)

$$x = \frac{l}{2}, \quad v = 0$$

$$\frac{l}{2} = \frac{l}{2} + \frac{F}{4K} + A\sin\phi$$

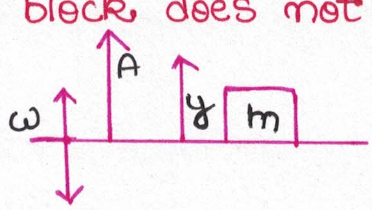
or $\sin\phi = \frac{-F}{4KA}$

$$\phi = -\pi/2, \quad A = F/4K$$

Coordinate from C.M.

$$x = \frac{l}{2} + \frac{F}{4K} + \frac{F}{4K}\left(\sin\left(\omega t - \frac{\pi}{2}\right)\right)$$

Que.) Find the max. amplitude that can be given to the floor so that the block does not lose contact with the floor.



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$y = A\sin\omega t$ $\quad (x_0 = 0)$

$a = -A\omega^2 \sin\omega t$

$N - Mg = -MA\omega^2 \sin\omega t$

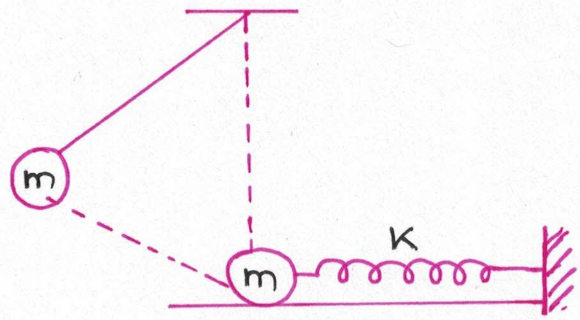
$N = M[g - A\omega^2 \sin\omega t]$

$N_{min} = M(g - A\omega^2) > 0$

$A < \dfrac{g}{\omega^2}$

N will be minimum when $\left(\omega t = \frac{\pi}{2}\right)$

Que.) Find time period of oscillation. If all collisions are perfectly elastic.



$T = \dfrac{T_S}{2} + \dfrac{T_P}{2}$

($T_S \rightarrow$ time pd. of spring
$T_P \rightarrow$ —"— of pendulum)

$T = \pi\sqrt{\dfrac{l}{g}} + \pi\sqrt{\dfrac{m}{K}}$